



Heriot-Watt University
Research Gateway

Illiquidity premium and expected stock returns in the UK: A new approach

Citation for published version:

Chen, J & Sherif, M 2016, 'Illiquidity premium and expected stock returns in the UK: A new approach', *Physica A: Statistical Mechanics and its Applications*, vol. 458, pp. 52–66.
<https://doi.org/10.1016/j.physa.2016.03.035>

Digital Object Identifier (DOI):

[10.1016/j.physa.2016.03.035](https://doi.org/10.1016/j.physa.2016.03.035)

Link:

[Link to publication record in Heriot-Watt Research Portal](#)

Document Version:

Peer reviewed version

Published In:

Physica A: Statistical Mechanics and its Applications

General rights

Copyright for the publications made accessible via Heriot-Watt Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy

Heriot-Watt University has made every reasonable effort to ensure that the content in Heriot-Watt Research Portal complies with UK legislation. If you believe that the public display of this file breaches copyright please contact open.access@hw.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.

Accepted Manuscript

Illiquidity premium and expected stock returns in the UK: A new approach

Jiaqi Chen, Mohamed Sherif

PII: S0378-4371(16)30033-4

DOI: <http://dx.doi.org/10.1016/j.physa.2016.03.035>

Reference: PHYSA 16999

To appear in: *Physica A*

Received date: 8 October 2015

Revised date: 26 January 2016



Please cite this article as: J. Chen, M. Sherif, Illiquidity premium and expected stock returns in the UK: A new approach, *Physica A* (2016), <http://dx.doi.org/10.1016/j.physa.2016.03.035>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

Title: "Illiquidity Premium and Expected Stock Returns in the UK: A new Approach"

Ref. No.: PHYSA-151558

- A single illiquidity measure is proposed to capture the multidimensionality of illiquidity.
- Both parametric and non-parametric methods are applied to investigate the relationship between illiquidity and stock returns in the UK.
- The inclusion of the illiquidity factor in the capital asset pricing model plays a significant role in explaining stock returns.
- The illiquidity-augmented capital asset pricing models yield a small distance error by using Hansen-Jagannathan non-parametric bound.

Illiquidity Premium and Expected Stock Returns in the UK: A new Approach

Jiaqi Chen^{*1} and Mohamed Sherif^{†2}

¹Accountancy, Finance and Economics Department, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom

²Accountancy, Finance and Economics Department, Heriot-Watt University, Edinburgh, United Kingdom & Cairo University Business School, Egypt

Abstract

This study examines the relative importance of liquidity risk for the time-series and cross-section of stock returns in the UK. We propose a simple way to capture the multidimensionality of illiquidity. Our analysis indicates that existing illiquidity measures have considerable asset specific components, which justifies our new approach. Further, we use an alternative test of the Amihud (2002) measure and parametric and non-parametric methods to investigate whether liquidity risk is priced in the UK. We find that the inclusion of the illiquidity factor in the capital asset pricing model plays a significant role in explaining the cross-sectional variation in stock returns, in particular with the Fama-French three-factor model. Further, using Hansen-Jagannathan non-parametric bounds, we find that the illiquidity-augmented capital asset pricing models yield a small distance error, other non-liquidity based models fail to yield economically plausible distance values. Our findings have important implications for managing the liquidity risk of equity portfolios.

Keywords: Asset pricing models; Multiple illiquidity measures; Hansen-Jagannathan test.

JEL Classification: E21; G12; G14; G15

^{*}E-mail: jc309@hw.ac.uk. We would like to thank Mustafa Caglayan for his constructive comments and suggestions. We are grateful to delegates in 2015 MFA Annual Conference in Chicago, BAFA 50th Annual Conference at London School of Economics as well as Recent Developments in Money, Macroeconomics & Finance Workshop at the University of Warwick. All remaining errors are ours.

[†]Corresponding author. Tel: +44 (0) 131 451 3681. E-mail: m.sherif@hw.ac.uk.

1. Introduction

The role of liquidity in asset pricing has grown rapidly over the past few years. A variety of studies have proposed different illiquidity measures as proxies for illiquidity by investors. However, although researchers are able to test whether the stock returns are statistically related to their illiquidity measures, their results generate conflicting impacts over stock returns. In other words, despite the increasing interest in the role of liquidity in equity markets in general, and asset pricing in particular, a universal definition for liquidity remains elusive, and the basic question of how to measure liquidity remains unsolved.¹ For example, Hasbrouck (2002) and Goyenko, Holden, and Trzcinka (2009) find that the measures are of a different quality themselves. They find that different measures have conflicting impact on stock returns: Amihud's price to volume measure is reported to have significant impact on stock returns but Pastor and Stambaugh's gamma is tested to have very little impact. In fact, if the empirical results are based solely on one particular measure, it is difficult to ascertain whether the results are driven by measure-specific components or by some common components of the measured illiquidity. Therefore, it is important to reconcile the conflict by collapsing all existing measures into one measure. Given the fact that strong evidence against the reliability of a single illiquidity measure exists, in this paper we adopt not only individual measures, but also construct a comprehensive illiquidity proxy. This illiquidity proxy is used across seven different measures and examines whether the pricing of liquidity risks varies amongst these measures. In particular, we adopt illiquidity measures introduced by Amihud (2002), Stambaugh (2003), zero-return measures proposed by Lesmond, Ogden, and Trzcinka (1999) and Liu (2006), Roll's (1984) effective bid-ask spread measure (Roll, 1984), the price-based spread measure of Corwin and Schultz (2012) and the effective tick measure from Goyenko et al. (2009). Consistent with Korajczyk and Sadka (2008) and Kim and Lee (2014), we find around 33% of the variation in illiquidity proxies is explained by the first principal component, which further suggests that systematic common components exist in illiquidity measures.

Our study contributes to understanding of the seemingly contradictory effects of illiquidity on asset pricing in several ways. It is generally the case that recent researchers have focused on new factors that contribute to tra-

¹Liquidity is a broad and elusive concept that generally denotes the ability to trade large quantities quickly, at low cost, and without moving the price. Financial literature indicates that rational investors who think they hold shares in exchange for lower returns with higher degree of liquidity they claim.

ditional asset pricing models. Indeed, Fama and French (2015) propose a brand new five-factor model while adopting indirect factor to denote liquidity. In contrast, in our approach we use UK data and examine the price of the common systematic components of illiquidity. There are indeed differences between the UK and the US environment in terms of trading and market structure. In the UK, all trading takes place on the London Stock Exchange (LSE) whereas in the US stocks are traded primarily on the Nasdaq and NYSE. In the US, trading on Nasdaq is based on order book driven while the NYSE uses a hybrid system. In the case of the UK, trading on the LSE is a mix of order book driven (SETS) and a hybrid quote/order book driven system. Furthermore, the UK is a bank-based system, which is more vulnerable to liquidity crunches than capital market-based system (US) because the first-order risk is bank solvency and the level of risk lies with financial institutions (Hardie and Maxfield, 2010). Since most studies on illiquidity premium and expected stock returns are predominantly based on US data, in this paper we seek to investigate if differences in market structure and liquidity characteristics of a country will lead to different results (Foran, Hutchinson, and O'Sullivan, 2014; Huang and Stoll, 2001). In this paper, we define 'illiquidity factor' as the spread return of equal-weighted portfolios $P10-P1$. These portfolios are constructed on the basis of the first principal component of the first seven illiquidity measures. Further, rather than the conventional parametric tests of asset pricing models, we use Hansen-Jagannathan distance to examine non-parametrically the level of errors associated with the liquidity capital asset pricing model (LCAPM). This helps shed light on these errors as an indication on the efficiency of the models.

This paper aims to provide answers to a number of questions. Firstly, based on existing illiquidity measures, is there a single illiquidity proxy that can significantly outperform other proxies with robust illiquidity premiums in asset pricing models in the UK? Secondly, does liquidity commonality exist in the UK? Thirdly, which liquidity-adjusted asset pricing model explains stock returns in the UK? Finally, do the results vary between parametric and non-parametric tests?

The remainder of the paper is set out as follows. Section 2 provides a brief review of the literature on the illiquidity framework. Section 3 provides details of the methodology and models we used to answer our questions. Section 4 presents the data and variable construction. Section 5 presents the empirical results and section 6 concludes.

2. Literature Review

What kinds of risk systematically drive stock prices? This question has prompted vast amounts of research and continues to exist as one of the main challenges in finance. The Sharpe-Lintner CAPM (1964) was the first attempt to answer this question by quantifying the risk which is attributable to general market fluctuations (Sharpe, 1964).

Yet, although the Sharpe-Lintner CAPM provides a theoretical framework to explain stock returns, the ability of the model to describe asset returns is weak. Indeed doubts regarding the empirical validity of the model are well established and it is both frequently rejected by data and also known to ignore some well documented anomalies, see *inter alia* (Black, 1972; Fama and MacBeth, 1973; Gibbons, 1982; Hyde and Sherif, 2005; Stambaugh, 1982). Traditional tests of the CAPM assume that the market portfolio is observable, expected returns are constant, and that assets' betas are stationary over a fixed period. Further, it measures risk by beta, which is a consequence of its questionable assumption of the existence of an equilibrium in which investors display mean-variance behaviour and requires the distribution of stock returns to be symmetrical.

The failure of the Sharpe-Lintner CAPM to capture the behaviour of the data and to measure a stock's or a portfolio's volatility has led to a number of different approaches that have attempted to address the limitations of the model. For instance, the three-factor model Fama and French (1993) and the Carhart (1997) model have received significant attention in empirical research. Whilst Fama and French (1993) demonstrate that asset prices are influenced not only by market systematic risk, but also the size and value factors, Carhart (1997) argues that momentum is an important risk factor which has not been priced in assets.

Recently, much attention has been given to market friction and in particular it has been widely argued that liquidity, see *inter alia* (Amihud, 2002; Bekaert, Harvey, and Lundblad, 2007; Chan and Faff, 2005; Chordia, Roll, and Subrahmanyam, 2001; Goyenko, Holden, and Trzcinka, 2009; Hasbrouck, 2002; Liu, 2006; Stambaugh, 2003), appears to be a suitable candidate for a priced state variable. For example, Lillo, Farmer, and Mantegna (2002) suggest that liquidity fluctuation is a permanent market impact. However, Bouchaud, Kockelkoren, and Potters (2006) argue that the impact power is transient and will decay in time. In fact, liquidity is often viewed as an important feature of the investment environment and the macro economy, and recent studies find that fluctuations in various measures of liquidity are correlated across assets. Furthermore, the importance

of liquidity on security returns has been confirmed by numerous previous empirical studies, which have thus established liquidity as a key consideration in investment decisions.

Nevertheless, using liquidity-based explanations are not straightforward. A difficulty in testing the liquidity-based explanation lies in the fact that stock liquidity is subjective concept and is very hard to measure. However, whilst liquidity is an elusive concept, most market participants agree that liquidity generally reflects the ability to buy or sell sufficient quantities quickly, at low trading cost, and without impacting the market price too much. Consequently, a vast number of measures have been used to approximate the extent to which a stock is illiquid or liquid. The first set of illiquidity measures have been based on stock daily returns or trading volume. Amihud (2002) proposes a simple and intuitive illiquidity measure, which is defined as the absolute daily return divided by daily trading volume. Acharya and Pedersen (2005) use the illiquidity proxy of Amihud (2002) and find evidence to support the model in the US market over the period 1962-1999. Elsewhere, Stambaugh (2003) have proposed a illiquidity measure called 'price sensitivity to order flow', which is based on return reversal due to heavy trading volume. Another illiquidity proxy is the turnover measured by daily share trading volume divided by the number of total shares outstanding.

The second set of illiquidity indicators are based exclusively on returns and provide a simple way of obtaining illiquidity proxy. For example, Liu (2006) proposes a trading volume-adjusted zero return measure and shows that the illiquidity measured by the proposed indicator is in fact priced in as far as the US market is concerned. It is worth noting that the zero return indicator is a number of zero return days scaled by the total available trading days in a given period. This measure indicates that on a day when trading cost is high, informed traders would not trade, resulting in zero return on that day. This measure is especially reliable in international finance research, as a high quality daily trading volume is not guaranteed (Bekaert et al., 2007; Lee, 2011). In addition, Lesmond et al. (1999) also propose a illiquidity measure based solely on daily returns. It was shown to be significantly correlated with the spread data and is used to show the illusionary aspect of momentum trading (Lesmond, Schill, and Zhou, 2004).

The third set of measures is based on return correlation, effective tick, and effective spread. Roll (1984) and Goyenko et al. (2009) suggest a proxy of spread based on the serial correlation of daily returns and effective spread. Also, in a recent study, Das and Hanouna (2010) create a measure of illiquidity based on 'run length', which totals the consecutive series of positive

and negative daily returns before the sign reverses. The authors further highlight that this particular illiquidity measure acts as a proxy for price impact.

2.1. Illiquidity Measures

Much literature suggests a number of proxies for illiquidity that are used as time-series conditioning variables. However, there are no agreed or final measures, and researchers have not yet reached an agreement regarding the optimal illiquidity proxy. In recent studies, Liu (2009) examine seven individual illiquidity measures. According to their findings, some proxies perform better than others in asset pricing models, which shows a more significant and robust illiquid premium. However, their results remain inconclusive regarding the most suitable illiquidity proxy. Arguably, a possible solution to find a suitable illiquidity proxy is an alternative method that extracts commonality of illiquidity risk. Indeed, recent researchers have implemented multiple illiquidity measures to gauge the robustness of their results. For example, Korajczyk and Sadka (2008) adopted multiple proxies and found that the illiquidity commonality exists among their measures. Similarly, Kim and Lee (2014) further found that the systematic common component of illiquidity measures risk in the US.

In this study we complement such approaches and add to the field by testing illiquidity individually. We form a composite index of illiquidity based on the common variation of a number of proxies for illiquidity, including turnover ratio, reversal measure of illiquidity, trading volume, bid-ask spread, effective spread, and number of zero return days.

1 Return/Value Ratio (Amihud, 2002)

The first illiquidity measure is the return to volume ratio proposed by Amihud (2002) to estimate illiquidity of stocks. This measure has been widely used in empirical literature because of its easiness of construction (Acharya and Pedersen, 2005). However, it is so far not clear that Amihud's measure would be priced, due to the compensation for price impact in comparison to other proxies which is something that requires further investigation.

Amihud defines illiquidity of stock i in time t as:

$$RV_i \equiv ILLIQ_t^i = \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{|R_t^i|}{V_t^i} \quad (1)$$

where R_t^i is the return on day d in month t , V_t^i is the dollar volume (in millions) on day d in month t , and $Days_t^i$ is the number of valid observation days in month t for stock i . In particular, V_t^i in this paper represents GB pound sterling (hereafter referred to as pound) volume (in millions) for the UK.

2 Reversal Measure of illiquidity (Stambaugh, 2003)

The reversal measure of illiquidity has been advocated by Stambaugh (2003). This measure reflects the return reversal after trading: the larger the volume, the larger the return reversal, and the larger the cost. Yet, one drawback of this measure is that it is time consuming in a real-time estimation. The measure is identified as:

$$r_{i,d+1,t} - r_{M,d+1,t} = \alpha_{i,t} + \beta_{i,t}r_{i,d,t} + \gamma_{i,t}sign(r_{i,d,t} - r_{M,d,t})dvol_{i,d,t} + \epsilon_{i,d,t}. \quad (2)$$

where $r_{i,d+1,t}$ is the return on stock i of day d at month t , $r_{M,d+1,t}$ is the market return (FTSE-All share value-weighted index return) on day d at month t , and $dvol_{i,d,t}$ is the pound trading volume (in million-pound unit). $\gamma_{i,t}$ is the coefficient of signed pound trading volume.

3 Zero Return (Lesmond et al., 1999)

Intuitively, when trading cost is higher than the benefit of trading, rational investors would choose not to trade (Lesmond et al., 1999). Therefore, we observe zero return for such days in this case. This measure is reported to be popular in international finance research, especially in emerging markets, where high-quality daily trading volume data are not available.

Lesmond et al. (1999) propose the zero return (ZR) illiquidity measure:

$$ZR_{i,t} = \frac{N_{i,t}}{T_t} \quad (3)$$

where T_t is the number of trading days at time t ; $N_{i,t}$ is the number of zero-return days of stock i in time t .

4 Turnover-Adjusted Zero-Return (Liu, 2006)

ZR measures can potentially lead to the same level of illiquidity for several stocks in multiple periods. In this case, Liu (2006) further proposed a turnover-adjusted zero-return measure, which is identified as:

$$LMx_{i,t} = \left\{ N_Z + \frac{1}{DF} \right\} \times \frac{21x}{N_x} \quad (4)$$

where N_Z is the number of zero-volume days in the previous x month; TV_x is the turnover over the previous x month, which is calculated as the sum of daily trading volume divided by the number of shares outstanding; N_x is the number of trading days in previous x months and DF is a deflator. Based on Liu (2006), we adopt the $LM12$ measure, which is based on the previous twelve months' data. Therefore, x is equal to twelve and we use the deflator of 11,000 as proposed by Liu (2006).

5 Bid-Ask Spread (Corwin and Schultz, 2012)

Amongst all of the proxies mentioned above, the bid-ask spread measure, in particular, has received extensive recognition by researchers. The data are widely available in real time and this measure can be calculated very quickly. However, the bid and ask quotes remain current only for a limited time periods. This is because the spread only measures the cost of executing a single trade of a certain size which requires complementary studies of other measures.

In a recent study, Corwin and Schultz (2012) developed the illiquidity measure from the ratio of daily high and low prices, excluding the volatility component. They define the spread estimator as:

$$S = \frac{2(e^k - 1)}{1 + e^k} \quad (5)$$

where K is identified as:

$$K = \frac{(\sqrt{2E\{\sum_{j=0}^1 [\ln(\frac{P_{t+j}^H}{P_{i+t}^L})]^2\}} - \sqrt{E\{\sum_{j=0}^1 [\ln(\frac{P_{t+j}^H}{P_{i+t}^L})]^2\}})}{(3 - 2\sqrt{2})} - \sqrt{\frac{[\ln(\frac{P_{t,t+1}^H}{P_{t,t+1}^L})]^2}{(3 - 2\sqrt{2})}} \quad (6)$$

where P_t^H and P_t^L are the high and low stock price at day t . Our monthly illiquidity measure CS is identified as the average daily estimated spread s in time t .

6 Bid-Ask Spread (Roll, 1984)

Roll (1984) proposes the effective spread based on the bid-ask spread:

$$RO_{i,t} = 2\sqrt{-COV(R_{i,d}, R_{i,d-1,t})} \quad (7)$$

where $R_{i,d}$ is the return of trading day d in month t and $R_{i,d-1,t}$ is the return of the previous trading day in the same month.

To make the possibility of positive covariance, we impose absolute values as suggested by (Lesmond, 2005). Consequently, in our study, Roll's measure is defined as:

$$RO_{i,t} = 2\sqrt{|COV(R_{i,d}, R_{i,d-1,t})|} \quad (8)$$

7 Effective Tick (ET)(Goyenko et al., 2009)

Finally, we employ the effective tick (ET) measure advocated by Goyenko et al. (2009). This measure is argued to be the simplest measure for all effective spreads. It is identified as:

$$ET = \frac{\sum_{j=1}^1 \gamma_j S_j}{\bar{P}_k} \quad (9)$$

We obtain S_j by using the decimal grid, which is an approach similar to that of the dollar grid proposed by Hagströmer, Hansson, and Nilsson (2011). In this case, the possible spreads are at £0.01, £0.05, £0.1, £0.2, £0.5 and £1. \bar{P}_k is the average daily prices in month k , and γ_j is defined as:

$$\gamma_j = \begin{cases} \text{Min}[\text{Max}\{U_j, 0\}, 1] & j = 1 \\ \text{Min}[\text{Max}\{U_j, 0\}, 1 - \sum_{k=1}^{j-1} \hat{\gamma}_k] & j = 2, 3, \dots, j \end{cases} \quad (10)$$

Based on

$$U_j = \begin{cases} 2F_j & j = 1 \\ 2F_j - F_{j-1}, & j = 2, 3, \dots, j-1 \\ F_j - F_{j-1} & j = j \end{cases} \quad (11)$$

where

$$F_j = \frac{N_j}{\sum_{j=1}^j N_j} \quad \text{for } j = 1, 2, J.$$

N_j is the number of trades on prices to the j spread using positive volume days.

3. Models & Methodology

3.1. Models

The analysis in this paper is based on the following standard capital asset pricing model:

$$R_t^p - R_t^f = \alpha_p + \beta_{p,MKT}MKT_t + \varepsilon_t^p \quad (12)$$

where R_t^p is the return of portfolio p in month t , R_t^f is the risk-free rate for month t , MKT_t , calculated as $(R_t^M - R_t^f)$ is the excess market portfolio return in month t and ε_t^p is the error term.

We also base our calculations in light of the fact that the Fama and French (1993) three-factor model is identified as:

$$R_t^p - R_t^f = \alpha_p + \beta_{p,MKT}MKT_t + \beta_{p,SMB}SMB_t + \beta_{p,HML}HML_t + \varepsilon_t^p \quad (13)$$

where SMB_t stands for size factor and HML_t is the value factor for time t .

Carhart (1997) further incorporated the momentum factor into the model as:

$$R_t^p - R_t^f = \alpha_p + \beta_{p,MKT}MKT_t + \beta_{p,SMB}SMB_t + \beta_{p,HML}HML_t + \beta_{p,MOM}MOM_t + \varepsilon_t^p \quad (14)$$

where MOM_t is the momentum factor.

In this paper, we incorporate illiquidity risk factor and apply the five factor model as:

$$R_t^p - R_t^f = \alpha_p + \beta_{p,MKT}MKT_t + \beta_{p,SMB}SMB_t + \beta_{p,HML}HML_t + \beta_{p,MOM}MOM_t + \beta_{p,L}L_t + \varepsilon_t^p \quad (15)$$

where L_t is the illiquidity factor.

3.2. Methodology

3.2.1. Principal Components Analysis (PCA)

Principal Component Analysis (*PCA*) is a powerful tool for analysing data as it has the ability when the data is in the form of a linear combination of optimally-weighted observed variables (Abdi and Williams, 2010). For a

given stock, we construct a correlation matrix of seven illiquidity measures and calculate the eigenvalue and eigen vector of the matrix. To compute scores on the first component extracted in a principal component analysis, the following model is employed:

$$COMP_1 = \beta_1(X_1) + \beta_2(X_2) + \beta_{1P}(X_P) \quad (16)$$

Or, in matrix notation:

$$COMP_1 = \beta_1^T X$$

where $COMP_1$ is the subject's score on principal component 1; $\beta_1(X_1)$ is the regression coefficient for the observed variable p , as used in creating the principal component 1; and X_p is the subject's score on the observed variable p .

The first principal component is calculated such that it accounts for the greatest possible variance in the data set. Clearly, it would be possible to make the variance of $COMP_1$ as wide as possible by choosing large values for the weights $\beta_{11}, \beta_{12}, \dots, \beta_{1p}$. To prevent this, weights are calculated with the constraint that their sum of squares is 1.

$$\beta_{11}^2 + \beta_{12}^2 + \beta_{13}^2 + \dots + \beta_{1p}^2 = 1 \quad (17)$$

The second principal component is calculated in the same way, with the condition that it is uncorrelated with the first principal component and that it accounts for the next highest possible variance.

$$COMP_2 = \beta_{21}(X_1) + \beta_{22}(X_2) + \beta_{2P}(X_P) \quad (18)$$

The calculation continues until a total of p principal components equal to the original number of variables has been generated. At this point, the sum of the variances of all of the principal components will equal the sum of the variances of all of the variables-that is, all of the original information has been explained.

3.2.2. Generalized Method OF Moment (GMM) & Fama and Mac-Beth (1973)

In our study we construct ten portfolios on the basis of common illiquidity and then test the joint significance of the ten portfolios' $\alpha(s)$. To

reduce heteroscedasticity and serial correlation problems, we estimate the $\alpha(s)$ using the systematic GMM.² For CAPM, we define r_t^x to be the 10×1 vector that contains excess returns of the ten portfolios, β_0 is the 10×1 vector for the constants, $B = [\beta_{MKT}]$ is the 10×1 matrix of portfolios' return sensitivities to market and $F_t = [MKT_t]$ is the 1×1 vector containing realisations of the factor. The standard CAPM can be identified as:

$$r_t^x = \beta_0 + BF_t + \varepsilon_t \quad (19)$$

To evaluate the model fit we use Hansen's J -test for over-identifying restrictions. The J -test provides a statistical test in cases where the moment conditions for a given model are significantly different from zero.

For the Fama-French three-factor model, $B = [\beta_{MKT}; \beta_{SMB}; \beta_{HML}]$ is the 10×3 matrix of portfolios' return sensitivities to market, size and value factor and $F_t = [MKT_t; SMB_t; HML_t]$ is the 3×1 vector. Similarly, in the Carhart four-factor model, $B = [\beta_{MKT}; \beta_{SMB}; \beta_{HML}; \beta_{MOM}]$ is the 10×4 matrix and $F_t = [MKT_t; SMB_t; HML_t; MOM_t]$ is the 4×1 vector.

We next perform the Fama and MacBeth (1973) two framework regressions to test the cross-sectional evidence of illiquidity factor in asset pricing models. When analysing cross-sectional data, the use of Fama-MacBeth regression has a number of advantages. First, it accommodates the dynamic explanatory variables. For the Fama-MacBeth regression the betas are estimated for a time period preceding the cross-section date which allows for time varying differences in the explanatory variables, whereas in other regressions, these variables are averaged out over the sample period and may lead to the loss of valuable information. Second, by running the cross-sectional regression and calculating what the standard errors are, they will then correct for cross-sectional correlations within the panel (Cochrane, 2001). Finally, the regression can also be extended to accommodate for additional risk features, beyond the beta (Campbell, Lo, MacKinlay, et al., 1997), and this is often useful if there are more risk factors to adhere to.

The first step of the Fama-MacBeth regression involves the estimation of betas after time-series regressions of the excess returns. Therefore, the illiquidity-augmented five-factor model becomes:

$$R_t^p - R_t^f = \alpha_p + \beta_{p,MKT}MKT_t + \beta_{p,SMB}SMB_t + \beta_{p,HML}HML_t + \beta_{p,MOM}MOM_t + \beta_{p,L}L_t + \varepsilon_t^p \quad (20)$$

²The OLS findings are available for readers upon request.

where R_t^p is the portfolio return p at time t , R_t^f is the monthly risk-free rate in month t , and MKT , SMB , HML , MOM and L are market return, size, value, momentum and illiquidity factors.

According to the same steps as previous studies, the first step of the regression estimates the time-series factors for each of the ten portfolios using 36 months rolling windows of 240 monthly observations. The second step estimates monthly cross-sectional regressions of the ten portfolio's excess returns on the betas that are estimated in the first step. Thus the model is identified as:

$$R_t^p - R_t^f = \lambda_0 + \lambda_{MKT}\hat{\beta}_{p,MKT} + \lambda_{SMB}\hat{\beta}_{p,SMB} + \lambda_{HML}\hat{\beta}_{p,HML} + \lambda_{MOM}\hat{\beta}_{p,MOM} + \lambda_L\hat{\beta}_{p,L} + \omega_t^p \quad (21)$$

where λ are the risk premium parameters with each beta. The hypothesis here is that the time-series average of the estimated coefficient λ_L is positive and statistically significant. This can be interpreted as showing the evidence that the illiquidity risk factor is priced.

3.2.3. Hansen-Jagannathan Distance

Rather than using formal statistical tests of identification and over-identification restrictions (statistical importance), it is possible to examine the model performance (economic importance) instead. The HJ distance measure is the mean square distance between the fitted values (\hat{m}) and the actual value m^* . The HJ minimum distance can then be presented as: $E(\hat{m} - m^*)$, where the expectations are estimated in practice using the sample averages. Since

$$m - m^* = E[m(\pi)X - 1]' E(XX')^{-1} X$$

The minimum distance is given by:

$$\left[E(m(\pi)X) - 1 \right]' E(XX')^{-1} \left[E(m(\pi)X) - 1 \right] \quad (22)$$

Let

$$g = [E(m(\pi)X) - 1]$$

and

$$W = E(XX')^{-1}$$

Then the minimum distance equals

$$\mathbf{g}'W\mathbf{g}$$

which is the Hansen J -test with a particular W .

Hansen and Jagannathan suggest comparing the pricing errors associated with the models in question by choosing each model's parameters θ to minimise the quadratic form:

$$h_t^{HJ} \equiv \mathbf{g}_T'(\theta) W_T^{-1} \mathbf{g}_T(\theta) \quad (23)$$

where $\mathbf{g}_T(\theta)$ is the sample average of pricing errors and W^{-1} is the sample second moment matrix of the N asset returns upon which the models are evaluated.

4. Data and Variable Construction

The data adopted in this study is monthly data and spans the period 1990-2012. The initial sample comprises the whole population of firms listed on the FTSE All-Share obtained from Thomson DataStream.³

For each index, we extract data including trading volume (turnover by volume); market value (share price multiplied by the number of ordinary shares in issue); return index (a theoretical growth in value of a share-holding over a specified period); and closing price. At the end of each month, the total number of shares outstanding, the return index, and the market value are obtained. Market to book value (market value of common equity divided by the balance sheet value of common equity in the company) is collected on an annual basis. We use the UK treasury bills 3-month yield rate as the risk free rate.

For the estimation of factor-asset pricing models, we construct size, value, and momentum risk factors. As for size, we sort all stocks based on their market capitalizations at month $t-1$ with a filter rule of 30% for portfolio formation. In other words, the value-weighted top 30% of stocks are allocated to the Big-size portfolio, whereas the value-weighted bottom 30% stocks are assigned to the Small-size portfolio. Therefore, the size (SMB) return is the difference between the returns of the small-size portfolio and the big-size portfolios at time t . Similarly to traditional empirical studies

³To avoid survivor-ship bias, this analysis covers not only presently listed stocks but also dead stocks. Dead stocks refer to those of firms that were de-listed at some point during our sample period.

applied in the UK market, we identify the value factor (HML) by obtaining the spread between monthly returns of the MSCI Value and MSCI Growth indices (Cuthbertson, Nitzsche, and O’Sullivan, 2008; Florackis, Gregoriou, and Kostakis, 2011).

For the momentum factor, we rank all stocks at month $t-1$ based on their returns from month $t-13$ to $t-2$. The equally-weighted top 30% of stocks are Winners and the bottom 30% are Losers. Thus the difference between monthly returns of Winner and Loser portfolios at time t is taken as the momentum factor (MOM)(Jegadeesh and Titman, 1993).

5. Empirical Results

We begin our analysis with the persistence of market illiquidity, as investors request a premium for bearing illiquidity only when the illiquidity shock is systematic and persistent (Acharya and Pedersen, 2005; Korajczyk and Sadka, 2008; Lee, 2011; Stambaugh, 2003). Table 1 reports the average monthly percentage returns, illiquidity, and other features for ten equally-weighted size portfolios. These are rebalanced each year based on the total market value of each stock at the end of the previous year. The existing literature reports a higher illiquidity with smaller stocks (Amihud, 2002; Amihud and Mendelson, 1986).

 INSERT Table 1 here

As seen in Table 1, illiquidity is generally higher for small stocks ($RV=4.6209$; $PS=0.1118$) than it is for large stocks ($RV=0.0012$; $PS=0.0001$). A similar pattern is also shown for LM , RO , and ET . The returns are higher for small stocks and we observe higher volatility for small stocks based on standard deviation. In line with Stambaugh (2003), Acharya and Pedersen (2005) and Korajczyk and Sadka (2008), we find that the equally-weighted average of stock illiquidity is highly persistent. Given the persistence of market illiquidity, similarly to Stambaugh (2003) and Acharya and Pedersen (2005), we construct the illiquidity innovations through $AR(2)$ as follows:

$$C_{M,t} \frac{MV_{M,t-1}}{MV_{M,1}} = \alpha_0 + \alpha_1 C_{M,t-1} \frac{MV_{M,t-1}}{MV_{M,1}} + \alpha_2 C_{M,t-2} \frac{MV_{M,t-1}}{MV_{M,1}} + \mu_{m,t} \quad (24)$$

where $C_{M,t}$ is the market aggregate illiquidity at month t ; and the residual $\mu_{m,t}$ is the illiquidity innovation. Notably, we scale PS and RV by the ratio

of the total market value by the end of the month $t-1$ to that in January 1990; $\frac{MV_{M,t-1}}{MV_{M,1}}$. This is in order to include only innovations in illiquidity, not the changes in time value of money. For other illiquidity measures, however, we apply the general $AR(2)$ regressions to find innovations:

$$C_{M,t} = \alpha_0 + \alpha_1 C_{M,t-1} + \alpha_2 C_{M,t-2} + \mu_{m,t} \quad (25)$$

The coefficients α_1 and α_2 are both significant. Further, the residuals do not display any serial correlation. Hence, we claim that $\mu_{m,t}$ accurately represents market illiquidity.⁴

Figure 1 shows the time-series plots of market aggregate illiquidity innovations for each measure and provides evidence that illiquidity innovations generally coincide with liquidity events in the timeline such as the Iraq invasion of Kuwait in 1990, the Asian crisis in 1997, the long term capital management crisis in 1998, and the financial crisis from 2007 to 2009. The fact that all seven measures jointly constitute liquidity-related events suggests the possibility that the individual proxy shares a common component of illiquidity.

 INSERT Figure 1 here

In Table 2 We report the correlations between market illiquidity proxies to examine whether illiquidity measures have a common component. We observe significant Pearson correlation tests in many cases. It measures the strength of the linear relationship between normally distributed variables. The highest correlation of 0.356 is shown between RV and RO and the lowest value among positive and significant correlations is 0.113 between ET and RV . However, RO is negatively correlated with most of the other measures in Table 2. The results imply that our illiquidity proxies, somehow, have systematic common components of illiquidity and this in turn justifies the use of the principal component analysis.

Next we use the principal component analysis to extract the common components of the seven illiquidity measures. As shown in Figure 2, the bar graph indicates the plot of the average eigenvalue proportions of seven principal components, as well as the plot of the cumulative proportions in the corresponding line graph. We find that the first principal component explains 33% of the whole variation over the seven illiquidity measures,

⁴Not reported but available from authors upon request.

which is coincidentally similar to findings reported in Korajczyk and Sadka (2008) and Kim and Lee (2014).

 INSERT Table 2 and Figure 2 here

Table 3 reports the descriptive statistics of illiquid portfolio performances from February 1990 to December 2012. We construct decile portfolios using the first principal component of illiquidity. In particular, at the end of month $t-1$, stocks are organised according to their first principal component extracted from the seven illiquid measures. Portfolio 1 ($P1$) includes stocks with the smallest ratio, whilst Portfolio 10 ($P10$) contains stocks with the highest values of illiquidity ratio and this excess return is only calculated for both $P1$ and $P10$. Portfolios are rebalanced on a monthly basis. Our empirical findings suggest that the average portfolio return increases from $P1$ to $P10$, though not monotonically. This pattern holds for equally weighted portfolios' returns but not for value weighted portfolios' returns. The level of this differential is about 16% per annum ($t=2.896$) for equally weighted portfolio returns. We also find no strong relationship between common illiquidity and market capitalization, nor do we find a clear correlation between common illiquidity and book to market ratio. Such results may have occurred due to our focus exclusively on the first principal component of the illiquidity measures. Nevertheless, certain features may continue to appear as measure-specific.⁵ However, our common illiquidity component clearly captures the change of the average β_{CAPM} associated with stocks, calculated by using a 36-month rolling window. The higher the illiquidity of the portfolio, the higher the beta we observe. The differential between $P10$ versus $P1$ beta is 0.287 ($t=8.933$).

 INSERT Table 3 here

Time-series Evidence of Risk-adjusted Returns

Table 4 presents the alphas of the value-weighted and equal-weighted portfolios sorted by the common component of the seven illiquidity ratios. For the principal component sorted equally-weighted portfolios, we find that

⁵Individual measure results are not reported but may be obtained upon request.

Jensen's alpha has generally increased across portfolios. Notably, most of the alphas across the portfolios (P1-P5) are with negative signs. Interestingly, *P10* has the highest and most significant alpha (CAPM) of 12.408%. Similar patterns hold for the Fama-French three-factor model and the Carhart four-factor model ($\alpha = 6.703\%$ and 8.480%) respectively. This suggests that portfolio returns increase with illiquidity. The last column presents the χ^2 statistic of the *Wald* test. The null hypothesis is that the alphas of the ten portfolios are jointly equal to zero. We fail to reject the null hypothesis. It is also worth noting that there is no certain pattern as we move from *P1* to *P10* and there are also insignificant premiums for value-weighted portfolios. This suggests that illiquidity premiums may be subject to the size factor. However, the χ^2 of the Wald test provides strong evidence against the null hypothesis.

 INSERT Table 4 here

Cross-sectional Evidences

As a robustness test, we further investigate the performance of the LCAPM in explaining the cross-section variations in stock returns.

Table 5 presents the estimated λ coefficients for the ten equally-weighted portfolios, and sorting is done based on the common component of illiquidity ratio in the UK. Panel *A* of Table 5 reports the unrestricted model with any value of λ_0 . The augmented illiquidity in Panel *A* is based on the first principal component. Our findings are supportive of the illiquidity augmented CAPM and the Fama-French model, as these specifications produce statistically positive, significant and economically sensible coefficients (premium λ_L), thereby offering a more valuable explanation of the data.

The estimated coefficient λ_L associated with CAPM and Fama-French (FF) models are significantly positive ($\lambda_{LCAPM}=5.76$ and $\lambda_{LFF}=5.19$) respectively, but the coefficient λ_L is insignificantly positive at ($\lambda_{LCARH}=10.4$) for the liquidity-augmented Carhart model. The penultimate column in Panel *A* reports the R^2 coefficients, and the last column reports the increase in R^2 coefficients after adding the illiquidity factor to the original models. The results show a good explanatory power, as the ΔR^2 has increased across all models, $\Delta R^2 = 0.041, 0.021$ and 0.035 for CAPM, FF and Carhart models respectively. In Panel *B* of Table 5, we present the estimated λ coefficients from the second framework cross-sectional regression of Fama-MacBeth. We

restrict λ_0 to be zero. The results yield similar findings as those in Panel A of Table 5, indicating strong and statistically significant coefficients for the liquidity-augmented CAPM and Fama-French model.

 INSERT Table 5 and Figure 3 here

As mentioned above (section 3), there are many reasonable measures that can be used to test the model specification. In section 3 we studied one of these measures which depends on a non-parametric function, the Hansen J test. However, Summers (1991) and Cochrane and Hansen (1992) claim that the GMM approach J test focuses too much on the specification of the model, and has too little focus on evaluating the accuracy of the underlying model. They argue that an increased focus on the accuracy of the model would help both reflect the purpose of understanding different types of behaviour and improve the ability of the model to make different types of predictions. To account for this criticism, we implement two alternatives advocated by Hansen and Jagannathan (1997) to assess the performance of the models.

Panel C of table 5 shows the robustness results of the Hansen-Jagannathan distance tests both with and without the illiquidity factor. We report both the principal component as of illiquidity. With the principal component illiquidity factor, we find that the error decreases from 0.304 to 0.198 and from 0.330 to 0.177 for the CAPM and Fama-French three-factor model respectively. Figure 4 also details the distance on the Hansen-Jagannathan bound, suggesting a significant empirical improvement for the liquidity Capital asset pricing models.

 INSERT Figure 4 here

Robustness Tests

In table 2, we report negative correlations of the RO measure with many other illiquidity measures, i.e. ‘Pastor and Stambaugh’s gamma’, ‘Corwin and Schultz’s spread’ and the ‘effective tick’. Such results indicate that the RO measure might differ more from other existing proxies. Therefore, we apply the robustness tests by removing the RO and estimate the new single illiquidity measure constructed by the principal component analysis. We report both parametric and non-parametric results in table 6.

 INSERT Table 6 here

Similar to our main findings, the new single illiquidity measure that is constructed by using the common component of the six illiquidity measures excluding RO proxy performs meaningful results. As for the parametric Fama-MacBeth estimations, we report positive and significant coefficient λ_L for CAPM and Fama-French models ($\lambda_{LCAPM}=6.423$ and $\lambda_{LFF}=7.331$). The coefficient λ_L is still insignificantly positive at ($\lambda_{LCARH}=5.657$ for the illiquidity-augmented Carhart model. The results show a slightly better explanatory power, but not a significant change in results as the ΔR^2 has increased across all models, $\Delta R^2 = 0.071$, 0.075 and 0.042 for CAPM, FF and Carhart models respectively. Panel C reports the results of Hansen-Jagannathan Distance using the new illiquidity measure generated from six illiquidity measures. The error decreases from 0.142 to 0.139, from 0.604 to 0.073, and from 0.619 to 0.168 for the CAPM, Fama-French three-factor model and Carhart four-factor model respectively. This implies that after they are augmented with illiquidity factor, the empirical asset pricing models significantly improve their pricing powers.

6. Conclusion

This paper examined the performance of the standard Sharpe-Lintner CAPM, the Fama-French three factor model (Fama and French, 1993), and the four factor model of Carhart (1997) both with, and without, the first component of multiple illiquidity measures. Further, the ability of the capital asset pricing models (CAPMs) to explain asset returns using solely individual illiquidity measures was also analysed. We used monthly UK data between 1990 and 2012.

Our initial investigation suggests that no individual illiquidity proxy out-performs the others, and further that our illiquidity proxies have a systematic common illiquidity component. Hence, we used the principal component analysis. According to the results of this analysis, the fact that seven measures jointly indicate liquidity-related events further suggests the possibility that the individual proxies share a common component of illiquidity. In addition, the correlations between market illiquidity proxies were considered in order to examine whether illiquidity measures share have a common component. Similarly to studies by Korajczyk and Sadka (2008)

and Kim and Lee (2014), the findings indicate that the first principal component explains 33% of the whole variation over the seven illiquidity measures in the UK. For illiquid portfolio and model performance, our findings are supportive of the illiquidity augmented CAPM and Fama-French model particularly with the portfolio of stocks with the highest illiquidity ratios *P10*. These specifications produce statistically positive, significant and economically coefficient estimates. For the non-parametric tests, we find that the illiquidity-augmented CAPM and Fama-French three-factor model yield a very small distance. These findings are supportive of the liquidity specification of the capital asset pricing models. Other non-liquidity CAPM models fail to yield economically plausible parameter values.

These findings have important implications for academic research into liquidity risk and for practical liquidity risk management alike. We contribute to the literature on liquidity risk by investigating the determinants of cross-sectional stock returns during liquidity crises. In addition, we analyse liquidity risk from a practical risk management standpoint. We show that abnormal stock performance during liquidity crises is, in part, predictable, and investors can construct portfolios of stocks that better withstand liquidity shocks. However, our results suggest that liquidity risk management comes at a cost of lower average returns during periods of relatively stable liquidity conditions.

Future research could investigate whether expected returns are related to stocks' sensitivities to fluctuations in other aspects of aggregate liquidity. It would also be useful to explore whether some form of systematic liquidity risk is priced in other financial markets, such as fixed income markets or international equity markets.

References

- Abdi, H., Williams, L. J., 2010. Principal component analysis. *Wiley Interdisciplinary Reviews: Computational Statistics* 2 (4), 433–459.
- Acharya, V. V., Pedersen, L. H., 2005. Asset pricing with liquidity risk. *Journal of Financial Economics* 77 (2), 375–410.
- Amihud, Y., 2002. Illiquidity and stock returns: Cross-section and time-series effects. *Journal of financial markets* 5 (1), 31–56.
- Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid-ask spread. *Journal of Financial Economics* 17 (2), 223–249.
- Bekaert, G., Harvey, C. R., Lundblad, C., 2007. Liquidity and expected returns: Lessons from emerging markets. *Review of Financial Studies* 20 (6), 1783–1831.
- Black, F., 1972. Capital market equilibrium with restricted borrowing. *Journal of Business*, 444–455.
- Bouchaud, J.-P., Kockelkoren, J., Potters, M., 2006. Random walks, liquidity molasses and critical response in financial markets. *Quantitative Finance* 6 (02), 115–123.
- Campbell, J. Y., Lo, A. W.-C., MacKinlay, A. C., et al., 1997. *The econometrics of financial markets*. Vol. 2. Princeton University press Princeton, NJ.
- Carhart, M. M., 1997. On persistence in mutual fund performance. *The Journal of Finance* 52 (1), 57–82.
- Chan, H. W., Faff, R. W., 2005. Asset pricing and the illiquidity premium. *The Financial Review* 40 (4), 429–458.
- Chordia, T., Roll, R., Subrahmanyam, A., 2001. Market liquidity and trading activity. *Journal of Finance*, 501–530.
- Cochrane, J. H., 2001. A rehabilitation of stochastic discount factor methodology. Tech. rep., National Bureau of Economic Research.
- Cochrane, J. H., Hansen, L. P., 1992. Asset pricing explorations for macroeconomics. In: *NBER Macroeconomics Annual 1992, Volume 7*. MIT Press, pp. 115–182.
- Corwin, S. A., Schultz, P., 2012. A simple way to estimate bid-ask spreads from daily high and low prices. *The Journal of Finance* 67 (2), 719–760.
- Cuthbertson, K., Nitzsche, D., O’Sullivan, N., 2008. Uk mutual fund performance: Skill or luck? *Journal of Empirical Finance* 15 (4), 613–634.
- Das, S. R., Hanouna, P., 2010. Run lengths and liquidity. *Annals of Operations Research* 176 (1), 127–152.
- Fama, E. F., French, K. R., 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33 (1), 3–56.

- Fama, E. F., French, K. R., 2015. A five-factor asset pricing model. *Journal of Financial Economics* 116 (1), 1–22.
- Fama, E. F., MacBeth, J. D., 1973. Risk, return, and equilibrium: Empirical tests. *The Journal of Political Economy*, 607–636.
- Florackis, C., Gregoriou, A., Kostakis, A., 2011. Trading frequency and asset pricing on the london stock exchange: Evidence from a new price impact ratio. *Journal of Banking and Finance* 35 (12), 3335–3350.
- Foran, J., Hutchinson, M. C., O’Sullivan, N., 2014. The asset pricing effects of uk market liquidity shocks: Evidence from tick data. *International Review of Financial Analysis* 32, 85–94.
- Gibbons, M. R., 1982. Multivariate tests of financial models: A new approach. *Journal of Financial Economics* 10 (1), 3–27.
- Goyenko, R. Y., Holden, C. W., Trzcinka, C. A., 2009. Do liquidity measures measure liquidity? *Journal of Financial Economics* 92 (2), 153–181.
- Hagströmer, B., Hansson, B., Nilsson, B., 2011. Conditional asset pricing with liquidity risk: The illiquidity premium. In: Working Paper.
- Hansen, L. P., Jagannathan, R., 1997. Assessing specification errors in stochastic discount factor models. *The Journal of Finance* 52 (2), 557–590.
- Hardie, I., Maxfield, S., 2010. What does the global financial crisis tell us about anglo-saxon financial capitalism? In: Workshop on the Financial Crisis, EMU and the Stability of Currencies and the financial System, University of Victoria.
- Hasbrouck, J., 2002. Trading costs and returns for us equities: The evidence from daily data. NYU Stern School Department of Finance Working Paper.
- Huang, R. D., Stoll, H. R., 2001. Tick size, bid-ask spreads, and market structure. *Journal of Financial and Quantitative Analysis* 36 (4), 503–522.
- Hyde, S., Sherif, M., 2005. Consumption asset pricing models: Evidence from the uk. *The Manchester School* 73 (3), 343–363.
- Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of Finance* 48 (1), 65–91.
- Kim, S.-H., Lee, K.-H., 2014. Pricing of liquidity risks: Evidence from multiple liquidity measures. *Journal of Empirical Finance* 25, 112–133.
- Korajczyk, R. A., Sadka, R., 2008. Pricing the commonality across alternative measures of liquidity. *Journal of Financial Economics* 87 (1), 45–72.
- Lee, K.-H., 2011. The world price of liquidity risk. *Journal of Financial*

- Economics 99 (1), 136–161.
- Lesmond, D. A., 2005. Liquidity of emerging markets. *Journal of Financial Economics* 77 (2), 411–452.
- Lesmond, D. A., Ogden, J. P., Trzcinka, C. A., 1999. A new estimate of transaction costs. *Review of Financial Studies* 12 (5), 1113–1141.
- Lesmond, D. A., Schill, M. J., Zhou, C., 2004. The illusory nature of momentum profits. *Journal of Financial Economics* 71 (2), 349–380.
- Lillo, F., Farmer, J. D., Mantegna, R. N., 2002. Single curve collapse of the price impact function for the new york stock exchange. Tech. rep., arXiv. org.
- Liu, W., 2006. A liquidity-augmented capital asset pricing model. *Journal of Financial Economics* 82 (3), 631–671.
- Liu, W., 2009. Liquidity and asset pricing: Evidence from daily data over 1926 to 2005. Nottingham University Business School Research Paper (2009-03).
- Roll, R., 1984. A simple implicit measure of the effective bid-ask spread in an efficient market. *The Journal of Finance* 39 (4), 1127–1139.
- Sharpe, W. F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk*. *The Journal of Finance* 19 (3), 425–442.
- Stambaugh, R. F., 1982. On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis. *Journal of Financial Economics* 10 (3), 237–268.
- Stambaugh, R. F., 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111 (3).
- Summers, L. H., 1991. The scientific illusion in empirical macroeconomics. *The Scandinavian Journal of Economics*, 129–148.

Table 1: Summary Statistics of Average Illiquidity by Size Portfolios

Portfolio	Return	RV	PS	ZR	LM12	RO	CS	ET	MarketCap	BTMV	St.Dev.
small	2.0939	4.6209	0.1118	0.0019	0.1332	0.0306	-0.0732	0.0042	1644.0017	1.1686	6.0109
2	1.4866	1.3653	0.0204	0.0013	0.1127	0.0268	-0.0624	0.0030	3145.8235	1.0700	5.5025
3	1.3134	0.9389	-0.0063	0.0012	0.1207	0.0282	-0.0627	0.0024	4731.2061	0.8846	5.4383
4	1.2193	0.5984	0.1049	0.0009	0.1116	0.0299	-0.0623	0.0018	6970.9691	0.8748	5.4623
5	1.2157	0.3619	0.0038	0.0009	0.1378	0.0285	-0.0620	0.0015	10570.0883	0.7935	5.2049
6	1.1545	0.1667	0.0032	0.0008	0.1403	0.0304	-0.0671	0.0013	15680.0039	0.7493	5.1954
7	0.8896	0.0832	0.0012	0.0006	0.1039	0.0325	-0.0707	0.0012	24416.1204	0.6775	5.5819
8	1.0252	0.0423	0.0072	0.0006	0.1154	0.0353	-0.0792	0.0008	42965.3213	0.6416	5.5332
9	0.9650	0.0058	0.0000	0.0005	0.1036	0.0374	-0.0889	0.0005	101928.4865	0.5394	5.2035
large	0.9222	0.0012	0.0000	0.0003	0.0783	0.0363	-0.0896	0.0003	781616.5530	0.4398	4.8251

Note: This table presents the average monthly percentage returns and illiquidity measures for 10 equally-weighted size UK portfolios. Portfolio's size is recalculated each year based on market value of shares at the end of the previous year. The illiquidity proxies are Amihud's illiquidity (RV), Pastor and Stambaugh's measure (PS), zero return (ZR), Liu's measure (LM12), Roll's measure (RO), the spread measure of Corwin and Schultz (CS) and effective tick (ET). Market cap is the market capitalization at the end of the previous year; BTMV is the book to market ratio; and St.Dev. is the standard deviation of portfolio return in the sample period.

Table 2: Pearson Correlation of Illiquidity Proxies

	RV	ZR	RO	PS	LM	CS	ET
RV	1						
ZR	0.060	1					
RO	0.356 ***	-0.048	1				
PS	0.042	0.082	-0.025	1			
LM	0.097	0.242 ***	0.064	0.047	1		
CS	-0.329***	0.093	-0.937***	0.037	0.052	1	
ET	0.113*	0.196***	-0.380 ***	0.117*	0.170 ***	0.297***	1

Note: See table 1 for illiquidity abbreviations. This table presents the correlation between proxies for market aggregate illiquidity measured by the equally-weighted average of stock illiquidity. *, **, and *** denote significance at the 10%, 5%, and 1% level respectively.

Table 3: Illiquid Portfolio Performances

Decile portfolios	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-P1	t-test
<i>EW Returns(%p.a.)</i>	1.055	-0.034	-0.233	-2.146	-1.872	8.487	8.040	9.848	9.907	17.122	16.067	2.896
<i>VW Returns(%p.a.)</i>	15.263	7.405	28.979	19.866	4.434	9.891	4.365	6.261	4.380	9.714	-5.549	-0.598
ILLIQ Ratio	-0.520	-0.337	-0.174	-0.025	0.096	0.316	0.629	0.932	1.244	1.987	2.507	17.313
MV(£m)*	64657.93	62875.07	68135.25	34739.21	40886.83	82550.36	90848.25	92813.58	91646.32	70198.99	5541.064	0.89
BTMV	0.398	0.294	0.254	0.155	0.193	0.617	0.731	0.712	0.702	0.684	0.286	9.017
β_{CAPM}	0.856	0.664	0.630	0.721	1.010	0.987	1.020	1.033	1.060	1.144	0.287	8.933

Note:., *P1* is the decile portfolio which has stocks with the lowest illiquidity ratio whereas *P10* has the stocks with the highest illiquidity ratio. *P10-P1* is the spread between *P10* and *P1*. *EW* returns are the annualized average monthly returns of equal weighted portfolios and *VW* returns account for the annualized monthly returns of value weighted portfolios. *MV* is the average market value of stocks in each of the portfolios in millions measured as the average of the share price times the number of shares outstanding. *BTMV* is the average ratio of the book value of shares divided by the market value in each portfolio. β_{CAPM} is the average stock beta in each portfolio using a 36-month sliding window. The *t-test* in the last column is the null hypothesis that the means are the same between *P10* and *P1*.

Table 4: Alphas Estimates of Value and Equally-Weighted Illiquid Portfolios

Decile portfolios	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	χ^2
Panel A:PCA Value-Weighted Portfolios											
$\alpha_{CAPM}(\% \text{ p.a.})$	11.400 (1.839)	4.564 (0.472)	25.888 (2.480)	16.360 (2.471)	0.857 (0.101)	6.009 (2.431)	0.599 (0.261)	2.138 (0.879)	0.337 (0.177)	4.931 (1.848)	7.182 (0.007)
$\alpha_{FF}(\% \text{ p.a.})$	5.409 (0.901)	-2.658 (-0.358)	21.106 (1.949)	11.578 (2.041)	-1.918 (-0.238)	6.450 (2.453)	0.257 (0.107)	2.345 (1.039)	-0.109 (-0.053)	3.496 (1.254)	4.526 (0.033)
$\alpha_{Carhart}(\% \text{ p.a.})$	4.575 (0.766)	-5.115 (-0.765)	18.749 (1.689)	9.927 (1.745)	-2.652 (-0.332)	6.692 (2.218)	0.461 (0.179)	3.443 (1.322)	0.464 (0.220)	5.407 (1.987)	3.993 (0.046)
Panel B:PCA Equally-Weighted Portfolios											
$\alpha_{CAPM}(\% \text{ p.a.})$	-0.960 (-0.344)	-2.409 (-0.787)	-1.695 (-0.684)	-2.905 (-2.115)	-3.527 (-1.935)	4.786 (2.210)	4.114 (1.776)	5.748 (2.744)	5.761 (2.515)	12.408 (3.677)	1.372 (0.241)
$\alpha_{FF}(\% \text{ p.a.})$	-2.883 (-1.078)	-4.337 (-1.425)	-2.901 (-1.138)	-3.471 (-2.564)	-5.276 (-2.945)	1.727 (1.076)	0.424 (0.374)	2.046 (1.734)	1.772 (1.487)	6.703 (3.405)	0.393 (0.531)
$\alpha_{Carhart}(\% \text{ p.a.})$	-1.761 (-0.710)	-3.071 (-1.095)	-2.282 (-0.940)	-3.511 (-2.528)	-4.080 (-2.370)	2.110 (1.312)	0.331 (0.271)	2.483 (2.053)	2.077 (1.655)	8.480 (4.333)	0.001 (0.986)

Note: $P1$ is the decile portfolio of stocks with the lowest illiquidity ratios; $P10$ is the portfolio of stocks with the highest illiquidity ratios. α is the annualized alpha estimated using the CAPM, Fama-French three-factor and Carhart four-factor models. T-statistics are reported in parentheses. The last column presents the χ^2 statistic of the *Wald* test. The null hypothesis is that the *alphas* of the ten portfolios are jointly equal to zero. The p-values are reported in the parentheses.

Table 5: Fama/MacBeth Estimates and Hansen-Jagannathan Distance

	λ_0	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{mom}	λ_L	R^2	ΔR^2
Panel A: PCA Unrestricted Model								
$CAPM_{ILLIQ}$	-0.434 (-2.729)	1.363 (4.338)				5.758 (3.015)	0.945	0.041
FF_{ILLIQ}	-0.354 (-2.622)	0.968 (1.434)	0.316 (0.267)	2.025 (1.608)		5.196 (1.976)	0.953	0.021
$CARHART_{ILLIQ}$	-0.131 (-0.916)	2.077 (1.948)	-2.155 (-1.018)	3.094 (2.716)	-5.263 (-2.492)	10.455 (1.077)	0.977	0.035
Panel B: PCA Restricted Model $\lambda = 0$								
$CAPM_{ILLIQ}$		-0.927 (-3.034)				9.338 (4.678)		
FF_{ILLIQ}		1.406 (1.968)	-0.1168 (-0.900)	2.739 (2.336)		10.973 (3.491)		
$CARHART_{ILLIQ}$		2.469 (2.603)	-3.149 (-1.759)	3.368 (2.929)	-6.031 (-3.358)	13.178 (1.537)		
Panel C: Hansen-Jagannathan Distance								
	CAPM	CAPM_ILLIQ	FF	FF_ILLIQ	Carhart	Carhart_ILLIQ		
δ	0.304	0.198	0.33	0.177	0.407	0.581		

Note: λ_i is the mean of risk premium coefficients λ_i using Fama and MacBeth (1973) method. The monthly cross-sectional regressions of ten equally-weighted portfolio return premiums are estimated using the risk factors of Fama and MacBeth (1973). *ILLIQ* is illiquidity factor of the CAPMs models. Panel A and B report systematic illiquidity factor augmented asset pricing models. T-statistics are reported in parentheses. The last column reports the increase in R-squared coefficient due to the addition of the illiquidity factor. Panel C reports the Hansen-Jagannathan distance. δ measures the distance error.

Table 6: Robustness Tests on Illiquidity Without RO

	λ_0	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{mom}	λ_L	R^2	ΔR^2
Panel A: PCA Unrestricted Model								
$CAPM_{ILLIQ}$	-0.357 (-2.090)	0.818 (2.291)				6.423 (4.057)	0.957	0.071
FF_{ILLIQ}	-0.138 (-0.69)	-0.565 (-0.806)	2.27 (2.656)	2.365 (2.445)		7.331 (3.677)	0.977	0.075
$CARHART_{ILLIQ}$	-0.119 (-0.576)	-0.414 (-0.583)	2.052 (2.506)	2.059 (2.266)	-2.098 (-1.918)	5.657 (1.003)	0.972	0.042
Panel B: PCA Restricted Model $\lambda = 0$								
	λ_0	λ_{mkt}	λ_{smb}	λ_{hml}	λ_{mom}	λ_L		
$CAPM_{ILLIQ}$		0.173 (0.493)				9.742 (4.922)		
FF_{ILLIQ}		-1.135 (-1.656)	2.953 (3.347)	2.815 (3.382)		8.155 (3.372)		
$CARHART_{ILLIQ}$		-0.859 (-1.232)	2.572 (3.061)	2.412 (3.127)	-1.803 (-1.611)	6.303 (0.750)		
Panel C: Hansen-Jagannathan Distance								
	CAPM	CAPM_ILLIQ	FF	FF_ILLIQ	Carhart	Carhart_ILLIQ		
δ	0.142	0.139	0.604	0.073	0.619	0.168		

Note: This table reports the robustness test results of both parametric and non-parametric estimations. The illiquidity factor is constructed without RO proxy proposed by Roll (1984).

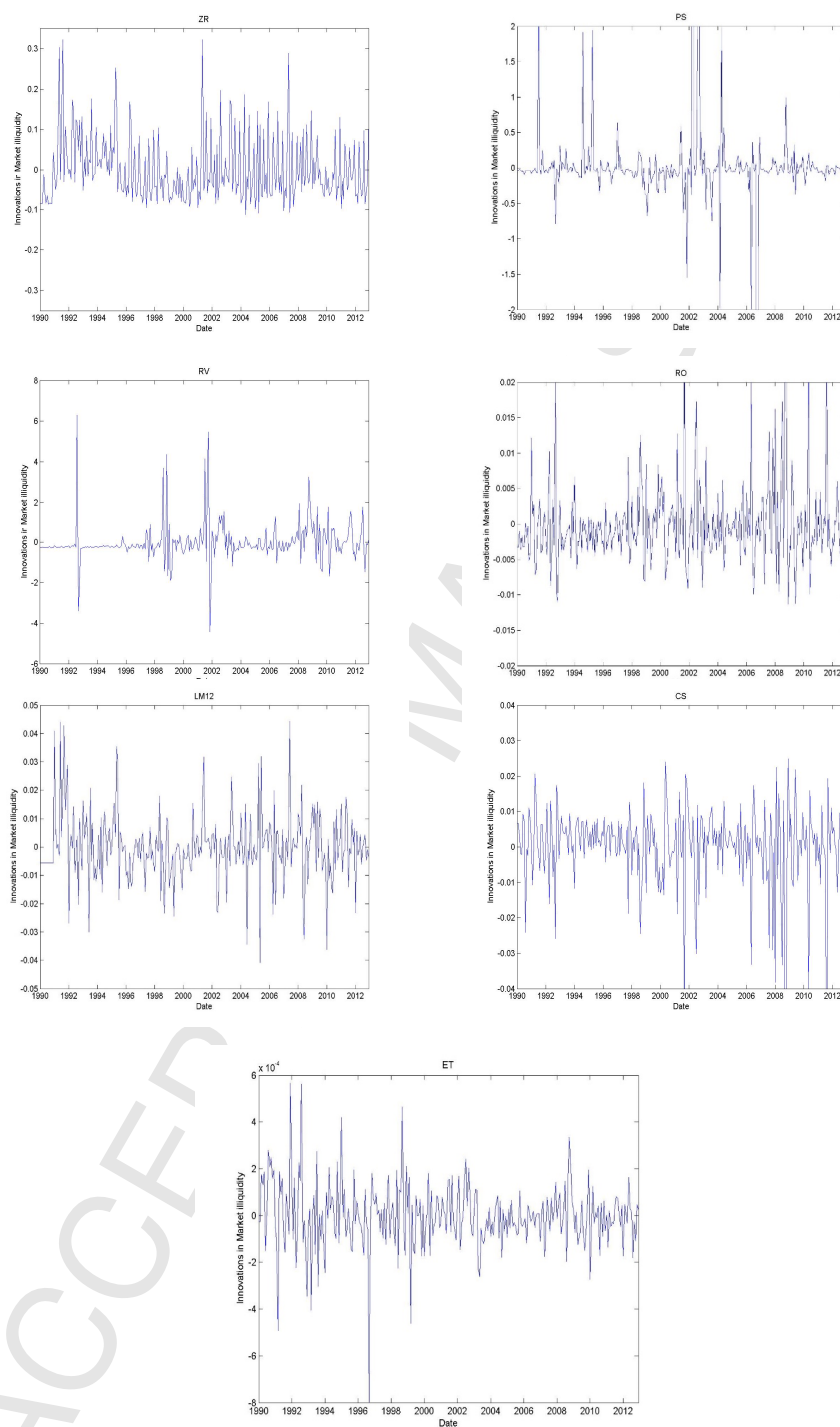


Figure 1: Innovations of Illiquidity Measures

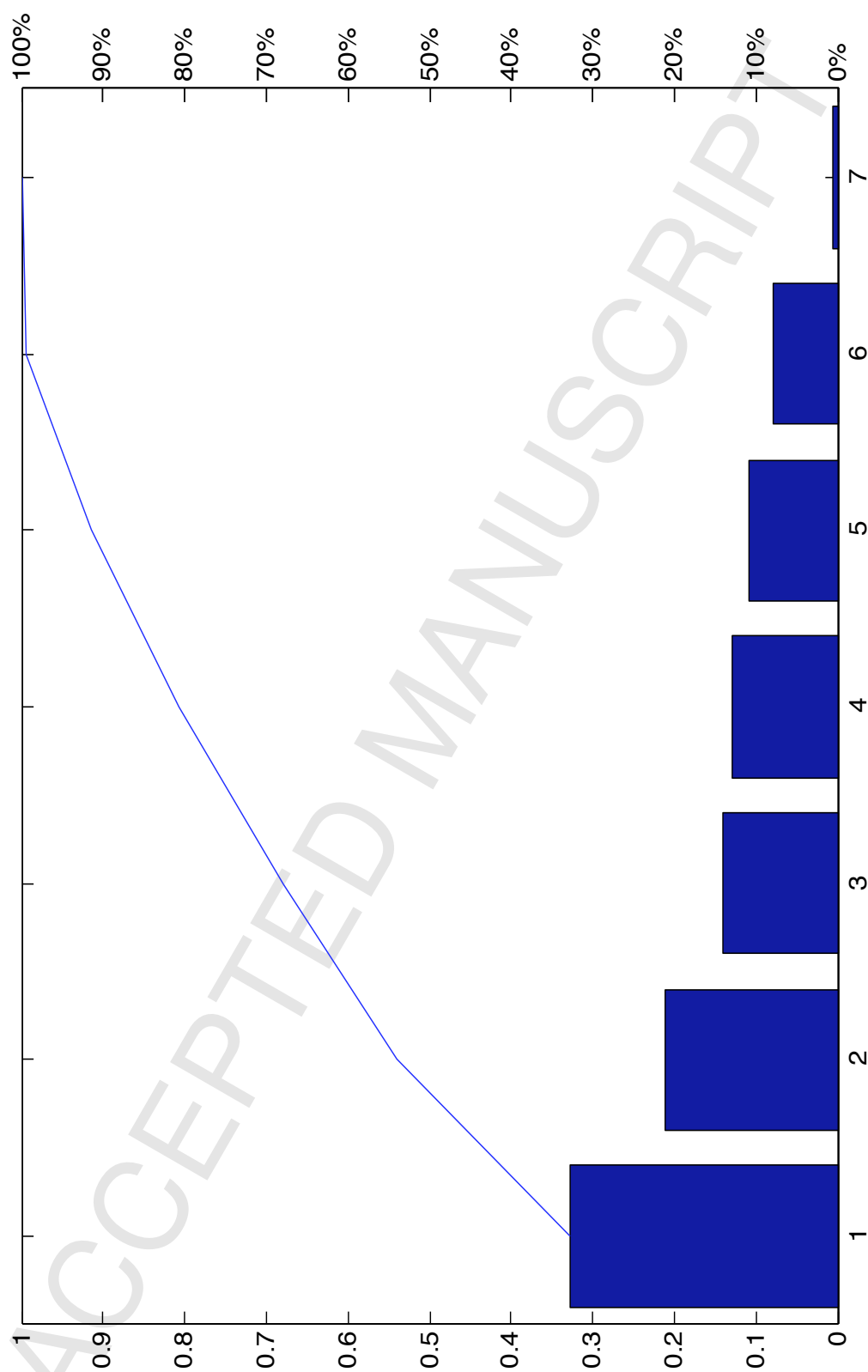


Figure 2: Eigenvalue proportion of principal components
bar:proportion line: cumulative proportion

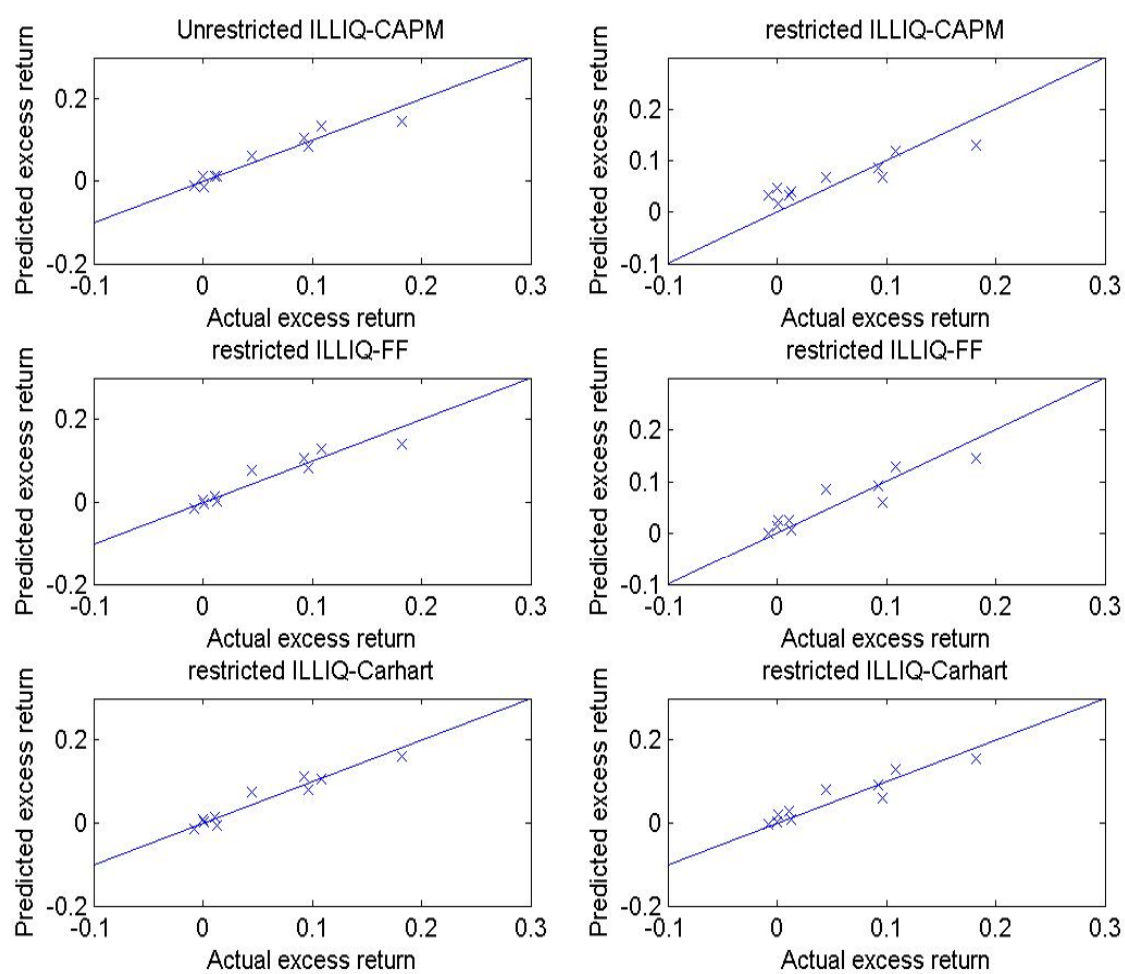


Figure 3: Fit of Illiquidity-augmented CAPM, Fama-French and Carhart Model

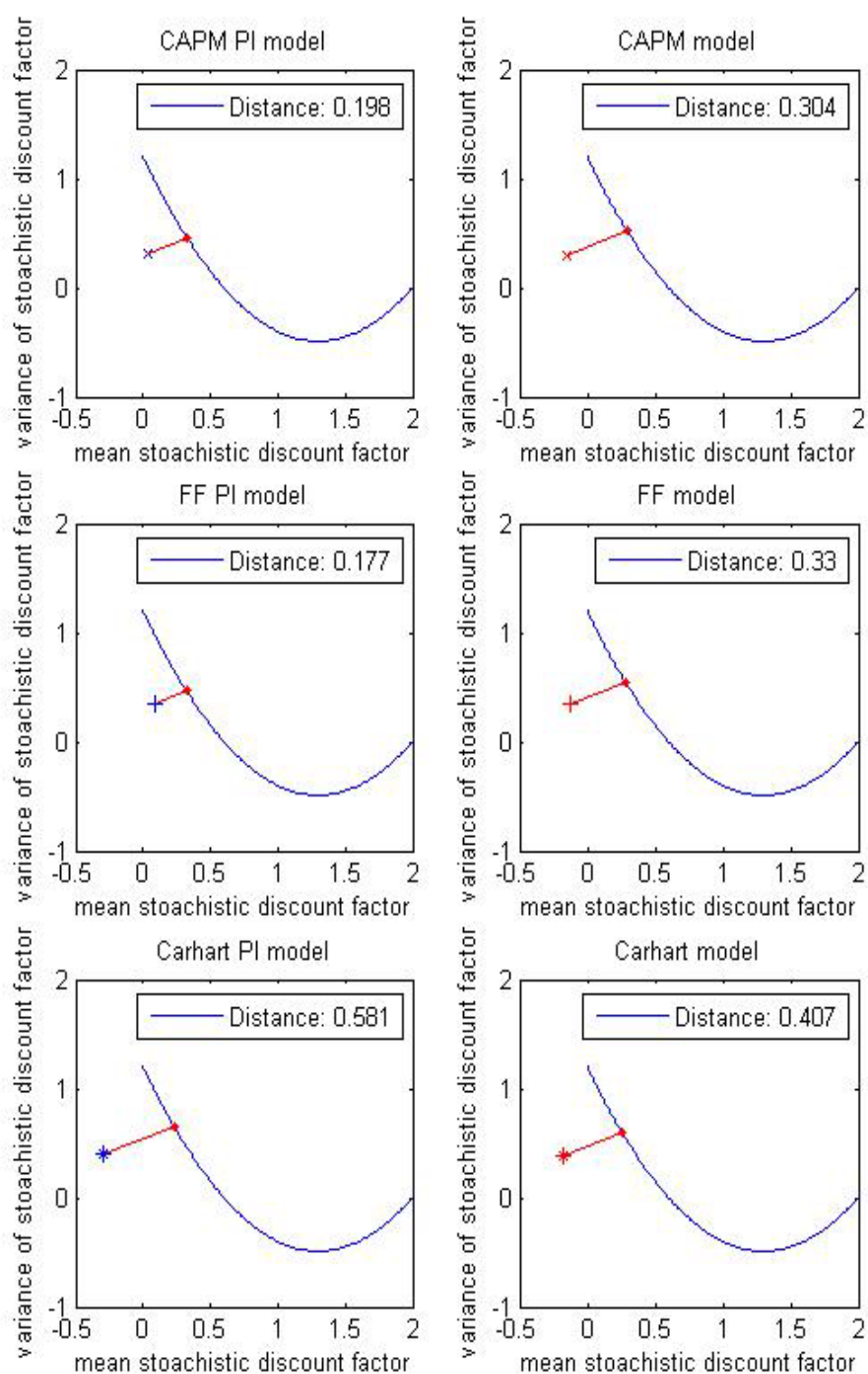


Figure 4: Hansen-Jagannathan Bound of Capital Asset Pricing Models